A GENERAL THEORY OF VALUE
Appendix Four: The Rules of *LIFE*

In *LIFE*, the number of neighboring cells that each cell must take into account before deciding its own next state is 8. (Each cell also takes its own (previous) state into account, but somewhat differently, as we shall see.) With $N = 8$ and each cell capable of being in one of two states, C_{pot} of the neighboorhood = $\log_2(2^N) = 8$ bits.[1] Somehow, eight bits of information have to be boiled down into one bit, because one bit of information is the most that a cell is permitted to display, both to us, as observers, and to its neighbors.[2]

To find R_1, we must compute the actual complexity-of-behavior that the rules will permit. We notice that rule(1), "if exactly two neighboring cells are on, stay in the present state (whatever that is, 'on' or 'off')," does not care *which two* neighboring cells are on. Nor does rule(2) care *which three*. And rule(3) just lumps all remaining conditions together. Here we see *LIFE* whittling and distilling eight bits of potential complexity down to.... Well, let us see.

All of *LIFE*'s rules entail *summing* the number of "on" cells without regard to order or position around the central cell. This summing procedure is in and of itself is very organizing. Figure A4.1 plots the number of ways, W(*k*), that 8 independent cells can be "on" or "off" *and* add up to *k* (where each "on" cell counts as 1 and each "off" cell as 0). Notice that the distribution is far from uniform, which would be the case if *every* possible way of arriving at $0 \leq k \leq 8$ was equally frequent or likely. This distribution would have no structure, no organization. The distribution *we* have with our summing procedure is, rather, a 'normal' or Gaussian one, with $k = 4$ as the most frequently produced sum.[3] Far from maximally complex, *this* distribution has structure (organization, in fact, to the tune of 1.9 bits).

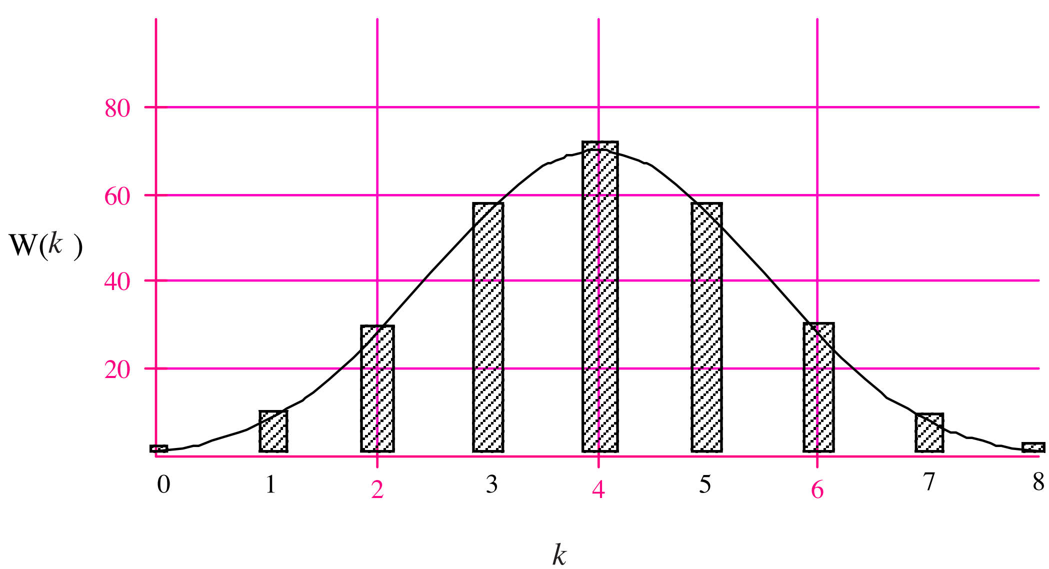

Figure A4.1 Frequency distribution of the number of ways, W(*k*), that 8 binary digits (= 1 or 0) can sum to *k* = 0, 1, 2,...8 when p("1") = p("0") = 0.5

Figure A4.1 depicts a situation in which all neighborhood cells are as likely to be found "on" as "off." But in *LIFE*, this is not the situation. In *LIFE*, the probability of a cell in an active pattern being "on" is around 0.2 and of being "off" 0.8, and this fact skews the symmetrical distribution of A4.1 considerably to the left. What we get with *LIFE*, rather, is shown in Figure A4.2.[4]

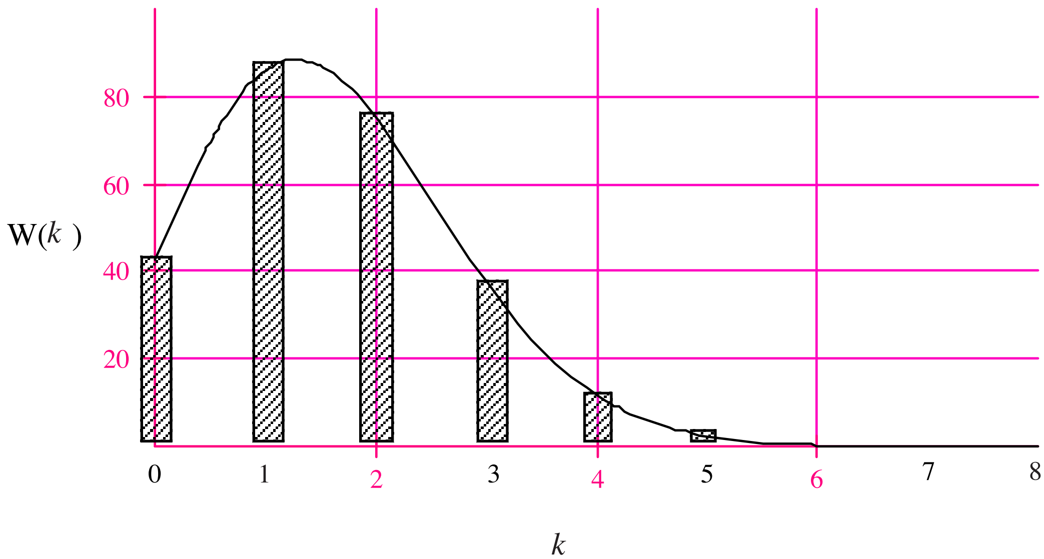

Figure A4.2 Frequency distribution of the number of ways, W(*k*), that 8 binary digits (= 1 or 0) can sum to *k* = 0, 1, 2,...8 when p("1") = 0.2 and p("0") = 0.8

The potential complexity, C_{pot}, of *LIFE*'s skewed distribution shown in Figure A4.2 is log(9) = 3.17 bits. (Why log(9) and not log(8)? Because *k*, since it includes zero, can have *nine* different values: 0, 1, 2....8. For the unskewed distribution shown in Figure A4.1, note that C_{pot} is also 3.17 bits).

Now applying Equation 1.4, we find that the actual complexity, C_{act}, of the *LIFE* distribution (think of it as a very biased 9-sided die) is 2.16 bits. This gives a magnitude for R_1 of 2.36 bits, and Ω = 2.24 bits. Notice that the closeness of C_{act} to R_1 keeps *LIFE* very much near the ridge of Ω, just as out theory predicts it should.

What remains to show is how *LIFE*'s rules keep p(*on*) around 0.2 and p(*off*) around 0.8 round after round of updates, "generation" after "generation."

The probability of exactly two neighborhood cells being "on" is 0.29. This is also the probability of rule(1) being applied. The probability of exactly three neighborhood cells being "on" is 0.15. This is also the probability of rule(2) being applied. All other neighborhood cell state permutations together have a probability of occurring of 0.56. This is the probability of rule(3) being applied. (For a listing of the rules, see Chapter Two, page...)

We now look at the transition probabilites p(*on*→*on*), p(*off*→*on*), p(*on*→*off*) and p(*off*→*off*). Applying 2-3 *LIFE* rules give us the following magnitudes

$$p(on \to on) = \overset{\text{rule(1)}}{0.2(0.29)} + \overset{\text{rule(2)}}{0.2(0.15)}$$

$$p(off \to on) = \overset{\text{rule(2)}}{0.8(0.15)}$$

$$p(on \to off) = \overset{\text{rule(3)}}{0.2(0.56)}$$

$$p(off \to off) = \overset{\text{rule(1)}}{0.8(0.29)} + \overset{\text{rule(3)}}{0.8(0.56)}$$

which means at every time step that

$$\begin{aligned} p(on) &= p(on \to on) + p(off \to on) \\ &= 0.2(0.29) + 0.2(0.15) + 0.8(0.15) \\ &= 0.21 \end{aligned}$$

$$\begin{aligned} p(off) &= p(on \to off) + p(off \to off) \\ &= 0.2(0.56) + 0.8(0.29) + 0.8(0.56) \\ &= .79 \end{aligned}$$

Near enough, no?

A question for mathematicians: are there an arbitary number of CA rule-sets, **R**, that have the property of preserving arbitrarily chosen but fixed ratios of p(*on*):p(*off*) in a finite CA region while actual configurations of "on" and "off" cells change with every time step? I suspect not.

In so-called "3-4 *LIFE*",[5] which is a little less productive of self-animating patterns than the original "2-3" version, the glider is as shown in Figure 2.14. It uses 8 or 9 "on" cells in a 6x6 grid rather than 5 "on" cells in a 5x5 grid, but the parameter settings in R_2 are adjusted to match.

NOTES to Appendix Four: "The Rules of *LIFE*"

[1] If every cell in *LIFE* treated its "earlier self" exactly like a neighbor, then C_{pot} would be equal to 9 bits.

[2] In CAs in general, an individual cell can be in one of S states, where $S \geq 2$. So can its N neighbors, $N > 1$. This means that the *neighborhood* of a single cell can be one of S^N different states.

Now S^N is a number which is always larger—and generally *much* larger—than S. This means that there can be no one-to-one mapping between neighborhood-states and cell-states, since in total number the latter always exceeds the former. What every CA's rule system must do then, in and for every cell, is reduce S^N down to S. This requires decision-making, "information processing," organization. Each cell must in some systematic manner distill the huge variety of neighboring-cell-state permutations down to the small number of states it can actually act out and display to other cells. And this must be true for all cells. Not only can no cell know what the others are "thinking," then, but no cell can be uniquely responsive to every combination of what its neighbors are "saying" (which is, as we have just seen, far less complex than what they are thinking). Indeed, no cell can exceed in sensitivity or expressiveness the sensitivity or expressiveness allowed to its neighbors without causing an escalating arms race that no cell can win, the avoidance of which is less a moral choice by CA designers than a mathematical necessity of CAs with one level of hierarchy.

I would argue that this necessity applies, within slightly broader limits, to all models of egalitarian/anarchic societies with the ideal of totally "(from-the-)bottom-up" self-organization by equally endowed citizens. As with cells n a CA, the complexity of behavior of individual members of a perfectly egalitarian society cannot come near the complexity of the behavior of an *ensemble* of similarly endowed and expressive neighbors. $S << S^N$. Every individual must collapse, ignore, or sift away at least $(\log S)(N - 1)$ bits of information coming from her or his social environment. Under these cicumstances, everyone is smarter "inside" than they look "outside" *by necessity*, and all complexity-of-expression greater than that which can be processed by neighbors-looking-at-their-neighbors-too is a waste.

A waste, that is, unless (1) there are *some* cells that have greater capacity for percieving and expressing complexity, and (2) *these* cells can communicate with each other on another level. For example, imagine starting with a CA like *LIFE* in which every cell can be in one of two states, black or white. Now let us endow some cells—say, every tenth in the grid in both directions—with the ability to display $2^8 = 256$ colors, *which* color being dependent on the permutations of state of their 8 always-black-or-white neighbors. (For their part, the color cell's black-or-white neighbors "see" half of the color cell's 256 colors *as black*, and half *as white*. They are, as it were, not only colorless but color-blind.) These new, color-expressing cells can be made to see and read each other and thus—using very much the same *LIFE* rules as the black-or-white cells use among themselves, and as they themselves use with their black-or-white neighbors—create a more complex-and-organized game of *LIFE*...create, in short, a class of cognoscenti, an elite. Goodbye egalitarianism.

Now let every *twentieth* cell be capable of displaying 1024 colors....

Insofar as CAs can model real life, real societies are clearly made up of "cells" that vary widely in their capacities for processing each other's information output. It is said that Bill Clinton knows 10,000 people by name; I know perhaps 100. Do simple folk, in this sense, stick with simple folk, and complex with complex?

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[3] The driving equation here is "8-choose-k" or $W(k) = 8!/[(8 - k)!k!]$, where "!" means "factorial," and k factorial means $(k - 0)(k - 1)(k - 2)....(k - [k - 1])$. For example, $4! = 4 \times 3 \times 2 \times 1 = 24$.

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[4] The driving equation here is $W(k) = (8!/[(8 - k)!k!])(0.2^k)(0.8^{8-k})(2^8)$. Thanks to Dr. Charles Friedman, Professor of Mathematics at the University of Texas at Austin, for deriving this formula.

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[5] In 3-4 *LIFE*, the parameters are set so that rule(1) calls for 3 cells "on," rule(2) calls for 4 cells "on," and rule(3) remains the same.
